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A NEW APPROACH TO IDENTIFY THE STIFFNESS MATRIX OF A COMPOSITE LATTICE STRUCTURES

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ABSTRACT

In this paper a new approach is presented to identify the stiffness matrix of composite lattice panels based on the superposition method per angle of helical ribs. The major aim of this study is to find out a simple approach in order to compute the general stiffness matrix of panels using appropriate relations and equations of composite mechanics. These panels which constitute layers of a laminate include three types of ribs, and these ribs are laminates with unidirectional layers, inside which fibers are arranged along the length of each rib. For showing the ability and performance of this technique an example is presented.

Key word: Composite Lattice Structures, Stiffness Matrix, Modulus of Elasticity

1. INTRODUCTION

Lattice structures have found growing applications in aerospace industries according to their high specific modulus and also low cost, extended studies have been made on these structures. These research works are generally conducted on cylindrical parts structures and are under axial loads and bending moments as well. Generally these structures are applied in the main body of aircrafts and interstage missile structures. Among these research works, V.V. Vasiliev *et al*. [1-2] have presented an integrated design, manufacturing and testing processes for high performance lattice structures made by continuous filament and wet winding of carbon and aramid epoxy composites and used as structural elements of airplane frames and space launching vehicles. In these papers, a lattice circular cylindrical structure with regular and dense system of ribs simulated by continuum models with ribs which are smeared over the structure surface, so to obtain its buckling behavior under axial loads. E. frulloni *et al*. [3] have presented a paper which is focused on the finite element modeling of the failure behavior of lattice composite hollow structures subjected to an external hydrostatic pressure. D. Slinchenko *et al*. [4] have presented a new approach to

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analysis of a lattice shell. In this approach, constitutive equations are developed and the expressions for components of stress and strain tensors are derived for the revolving shells with different lattice patterns. In fact according to increasing demand by industries and also various applications of laminates, the use of laminates which contain lattice layers to decrease the weight of the structure seems to be a prerequisite not just for cylindrical shells but for every shape as well.

In this study, a lattice layer has been analyzed via a simple method. This approach has been tested based on calculation of the stiffness matrix of a lattice layer as a laminate ply with superposition method, and then with the aid of this matrix, modulus of elasticity $\left(E_x, E_y, G_{xy}, v_{xy}\right)$ as a function of rib's angle and other variables are examined.

2. THE STRUCTURE OF LATTICE LAYERS AT JOINT POINTS

Lattice panels include three types of ribs. One of the ribs is arranged in horizontal state and the other two in helical states. The directions of helical ribs are symmetric around vertical axis. The arrangement type of ribs in each layer, however, is an important issue. In each rib, a filler foam layer (or the same resin used in composite layers) is placed between each two composite layers as shown in *Fig*. (1). This foam contains compound materials with low density. These layers are arranged purposely to allow continuity of composite layers at joint points, so that at each node, composite layers are continued and foam layers are discontinuous.

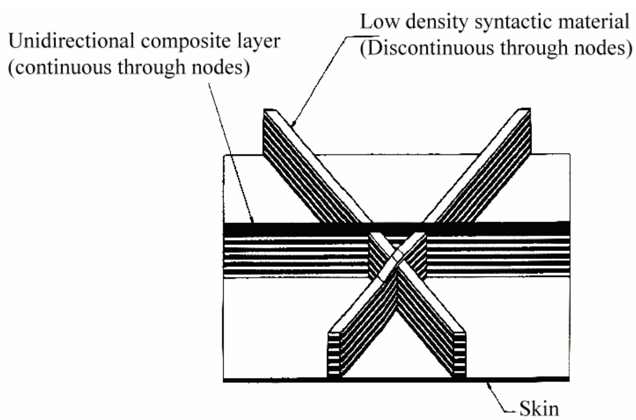


Fig. 1. Joint points of ribs [5]

The important result is that joint node which is placed between ribs, individually, can stand no moment as for this point we do not consider significant mechanical properties shared with filler foam. Paying specific attention to this point is of outmost importance when using superposition method, because in this approach the effect of each rib against applied loads is calculated separately which would finally combine together. This means that if joint points of ribs bear any moment, the sum effect of ribs is not equal with the sum of applied loads and as a result our method would not lead to an ideal situation.

3. GENERAL ANALYSIS

Relations between stresses and strains in a laminate plan are stated by $[A]$, $[B]$ and $[D]$ matrices as we know it from the mechanics of a composite laminated plan. *Eq*. (1) expresses relationship between the forces and applied moments with strains and plan curvatures calculated via stiffness matrix of each layer according to relations available in composite mechanics:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa \end{bmatrix} \tag{1}$$

Relations between stress and strain in each layer are stated by stiffness matrix of that layer as we already know it from mechanics of a single layer:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_s \end{bmatrix}_k = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_s \end{bmatrix} \tag{2}$$

In this equation, the constants of stiffness matrix of each layer are computed according to the characteristics of that layer as well as using equations available in references related to composites mechanics.

According to the above discussed issues, the only feature remained necessary for calculation of stiffness matrix of a composite laminate which also considers lattice layers is the way the stiffness matrix of lattice layers is computed.

4. DERIVING OF STIFFNESS MATRIX OF A LATTICE LAYER

A lattice plan includes three groups of ribs which are arranged in three directions. These directions are specified by angles of ribs with axis (x), and for defining mechanical properties of a plan, other variables are needed. These variables which are shown in *Fig*. (2) include.

In order to use superposition method for mechanical analysis of lattice layers, some points and assumptions are needed to derive the required equations state by state:

- Considering the superposition method, a lattice layer is assumed to have just one group of ribs and the effect of other ribs in each state is not taken into account. In each state, the resulted plane bears loads just along the ribs. It means that parallel ribs do not transfer loads to each other.
- It is assumed that lattice layer is a continuous and conventional single layer with unidirectional fiber. In this assumption, the direction of ribs selected above is considered as direction (1).

Note: Direction (2) is normal to direction (1) and should not be mistaken with direction of ribs which are symmetric with ribs of direction (1).

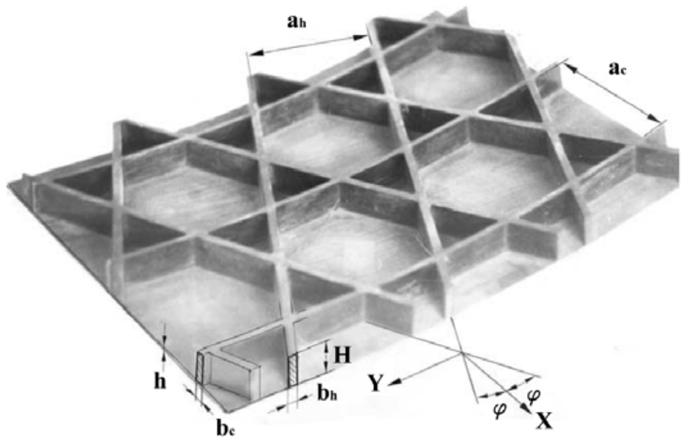


Fig. 2. Geometrical variables of a lattice plane: H = Height of cross-section of ribs (thickness of lattice layer), b_h = Thickness of cross-section of helical ribs, b_c = Thickness of cross-section of horizontal rib, a_h = Distance between helical ribs, a_c = Distance between horizontal ribs, φ = Angle of helical ribs with axis (x)

The first result from above assumptions is that according to discontinuity of constituents of a single layer which contains only one collection of ribs, the strain in direction (1) has no effect on strain in direction (2). Hence poisson's ratio for such a layer is zero in direction 1 and 2 $(v_{12} = v_{21} = 0)$.

Now, in order to compute stiffness matrix of a desired layer, related to (*x*) and (y) axes, the amount of E_1, E_2 and G_{12} are needed to be computed.

4.1 Computation of E_1

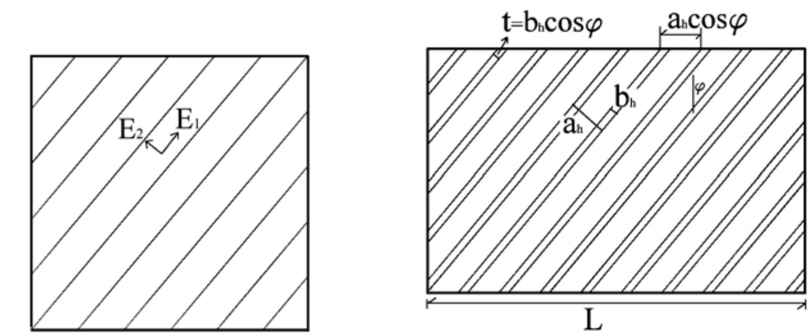
According to section (2) assumptions, the ribs are considered as unidirectional fibers and arranged into a resin (empty space in this case) in order to compute E_1 as shown in *Fig*. (3). According to micromechanical equations, for computing E_1:

$$E_1 = E_{rib_h} V_{rib} + E_{void} V_{void} \tag{3}$$

where:

E_{rib_h} = Elastic constant of a composite plane of which the rib is made. This parameter is obtained from equation below:

$$E_{rib_h} = E_h = E_f V_f + E_m V_m \tag{4}$$



(A) Directional of longitudinal and transverse modules *(B) Variables*

Fig. 3 A lattice plane, just one collection of ribs

E_{voids} = Elastic constant of empty space that is zero.

$$E_{void} = 0 \tag{5}$$

V_{void} and V_{rib} = Volume fraction of ribs and empty space are obtained as below, which according to *Fig*. (3):

$$V_{rib} = \frac{A_{rib}}{A_{TOT}} = \frac{nb_h \, cos}{na_h \, cos} = \frac{b_h}{a_h} \tag{6}$$

According to above equations, it is resulted that:

$$E_1 = E_h \frac{b_h}{a_h} \tag{7}$$

On the other hand, according to assumptions (parallel ribs have no effect on each other), it is obvious that E_2 and G_{12} are zero for such a plane:

$$E_2 = G_{12} = 0 \tag{8}$$

According to obtained results, stiffness matrix of desired plan is obtained as below:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} = \frac{E_h \, b_h / a_h}{1 - 0 \times 0} = E_h \frac{b_h}{a_h} \tag{9}$$

$$Q_{11} = Q_{22} = Q_{66} = 0$$

Now stiffness matrix of plan, relative to (x) and (y) axes can be derived via continuum mechanics transforming equations.

4.2 Calculation stiffness matrix in each state and final results

Using result for a plan which contains just one group of ribs in direction (1) and according to this point that joint points between ribs bear no moment, final stiffness matrix of lattice layer can be achieved via superposition method.

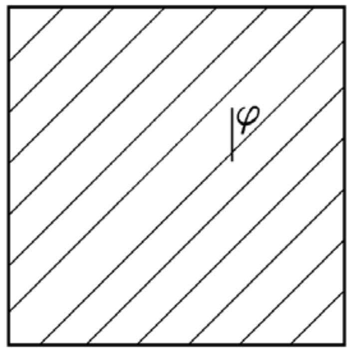


Fig. 4. Ribs with ($+\varphi$) angle

After computing layer stiffness matrix in three various states, so that in each state there is just one group of ribs, they are added together and general stiffness matrix is obtained. These states and resulted matrices are:

1-Plane just contains ribs which have a ($+\varphi$) angle with (x) axis. In this state according to *Eq*. (9), plan stiffness matrix is:

$$\left([Q]_{12}\right)_{+} = \begin{bmatrix} E_h\, b_h / a_h & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (10)$$

with the use of transforming Equations, $\left([Q]_{xy}\right)_{+}$ matrix is obtained:

$$\begin{bmatrix} E_h b_h / a_h \cos^4 & E_h b_h / a_h \sin^2 \cos^2 & E_h b_h / a_h \cos^3 \sin \\ Q_{21}=Q_{12} & E_h b_h / a_h \sin^4 & E_h b_h / a_h \cos \sin^3 \\ Q_{61}=Q_{16} & Q_{62}=Q_{26} & E_h b_h / a_h \sin^2 \cos^2 \end{bmatrix} \qquad (11)$$

2- Plane just contains ribs which have $(-\varphi)$ angle with (x) axis. Resulted matrix for this state is similar to that of previous state. But in this matrix Q_{16}, Q_{61}, Q_{26} and Q_{62} have a similar but negative quantity because angle of ribs has minus sign:

$$\begin{aligned} (Q_{16} = Q_{61})_{-\varphi} &= -(Q_{16} = Q_{61})_{\varphi} \\ (Q_{62} = Q_{26})_{-\varphi} &= -(Q_{62} = Q_{26})_{\varphi} \end{aligned} \qquad (12)$$

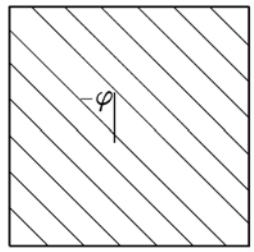


Fig. 5. Ribs with $(-\varphi)$ angle

3- Plane contains only horizontal ribs $(\varphi = \pi/2)$.

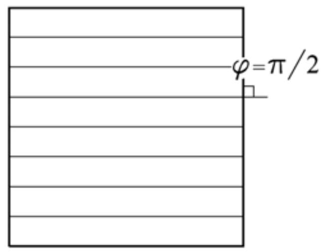


Fig. 6. Horizontal ribs with an $(\varphi = \pi/2)$ angle

According to the method used to derive *Eq*. (7):

$$E_1 = E_{rib_c} \frac{b_c}{a_c} \tag{13}$$

According to the resulting amounts and with the use of equations for constants of stiffness, matrix $([Q]_{12})_{\pi/2}$ is obtained:

$$([Q]_{12})_{\pi/2} = \begin{bmatrix} E_c\, b_c/a_c & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{14}$$

Now with the use of transforming equations $([Q]_{xy})_{\pi/2}$ is obtained:

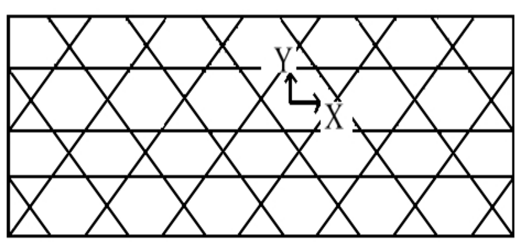


Fig. 7. A lattice layer

$$\left([Q]_{xy}\right)_{\pi/2}=\begin{bmatrix}0 & 0 & 0\\ 0 & E_c\dfrac{b_c}{a_c}sin^4(\pi/2) & 0\\ 0 & 0 & 0\end{bmatrix} \tag{15}$$

According to three resulted matrices and via superposition method, general stiffness matrix of lattice layer is obtained by adding the matrices together:

$$\left([Q]_{lattice}\right)_{xy}=\left([Q]_{xy}\right)_{+}+\left([Q]_{xy}\right)_{-}+\left([Q]_{xy}\right)_{\pi/2} \tag{16}$$

So, constants of $\left([Q]_{Lattice}\right)_{xy}$ matrix are:

$$\begin{bmatrix}2E_h b_h/a_h\cos^4\varphi & 2E_h b_h/a_h\sin^2\varphi\cos^2\varphi & 0\\ Q_{21}=Q_{12} & E_h b_h/a_h\sin^4\varphi+E_c b_c/a_c & 0\\ 0 & 0 & 2E_h b_h/a_h\sin^2\varphi\cos^2\varphi\end{bmatrix} \tag{17}$$

The resulted matrix is the stiffness matrix of a lattice layer in (*x*) and (y) general co ordinations as shown in *Fig*. (7).

Now $[A]$, $[B]$ and $[D]$ matrices for a laminate which contains lattice layers, are computable by equations used for calculation of these matrices [6].

5. VARYING ELASTICITY MODULUS OF A LATTICE LAYER AS FUNCTION OF HELICAL RIBS ANGLE

One of the applications of a generally orthotropic lamina stiffness matrix is to determine the stiffness behavior of the considered layer. The properties which generally include modulus of elasticity and also strains ratio in (*x*) and (y) directions, can be computed by compliance matrix as presented in Table 1.

Table 1 - Mechanical properties of the composite material

Properties	**Symbols**	**Magnitudes**
Fiber volume fraction	V_f (%)	62
Longitudinal modulus	E_1 (GPa)	88
Transverse modulus	E_2 (GPa)	8.8
Shear modulus	G_{12} (GPa)	2.8
Poisson's ratio	ν_{12}	0.28

For lattice layers the easiest method to attain compliance matrix is to compute the inverse of its stiffness matrix:

$$\left([S]_{Lattice}\right)_{xy}=\left[\left([Q]_{Lattice}\right)_{xy}\right]^{-1} \tag{18}$$

In order to compute modulus of elasticity in (x) and (y) directions and Poisson's ratio, the defined equations of composite mechanics are used [5], therefore:

$$E_x = \left(1/\bar{S}_{11}\right) \quad E_y = \left(1/\bar{S}_{22}\right) \quad G_{xy} = \left(1/\bar{S}_{33}\right) \tag{19}$$

$$\nu_{xy} = \left(-\bar{S}_{21}/\bar{S}_{11}\right) \quad \nu_{yx} = \left(-\bar{S}_{12}/\bar{S}_{22}\right)$$

In order to explain that how the mentioned properties depend on angle of helical ribs, elasticity modulus and Poisson's ratio (in x and y direction) are computed for a lattice layer which its ribs (helical and horizontal) are made of graphite/epoxy composite. The mechanical properties of this composite are shown in *Table* (1). In this study, the angle of ribs is varying between 0° to 90° and their geometrical variables (*Fig*. 2) are assumed as bellow:

$$a_h = a_c = 2\ mm \quad b_h = b_c = 6\ cm$$

Variation curves of elasticity modulus and Poisson's ratio have shown in *Fig*. (8–9). As it is shown, these properties are a function of helical ribs angle where geometrical variables are defined.

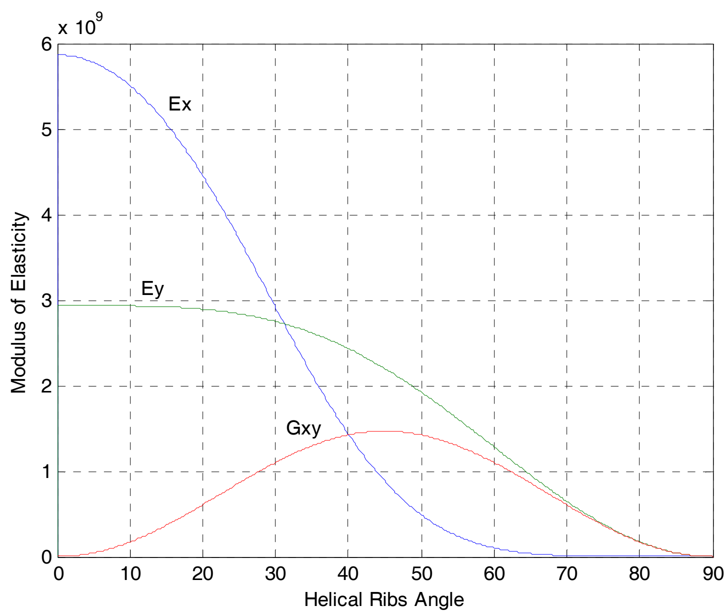


Fig. 8. Variation of elasticity modulus (GPa) versus (φ)

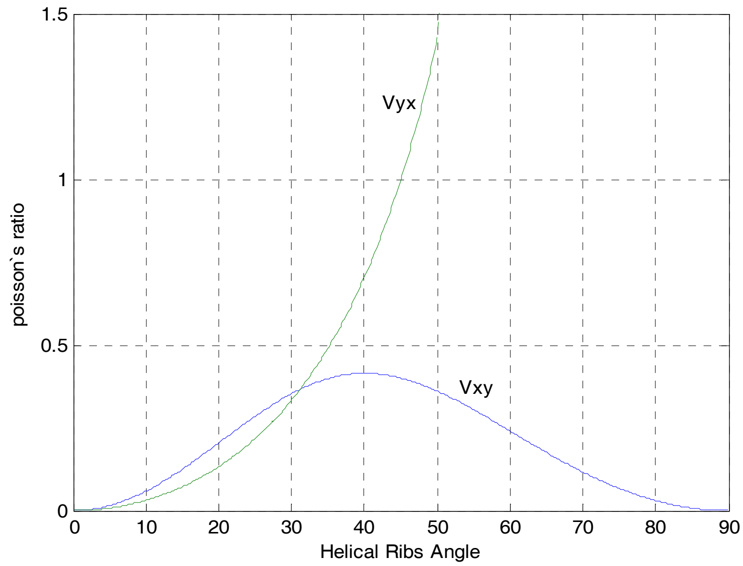


Fig. 9. Variation of Poisson's ratio versus (φ)

6. RESULTS AND DISCUSSIONS

As it is shown in *Fig*. (8), E_x curve has a descending state versus angle (φ) variations and equal to zero at $(\varphi = \pi/2)$. This state is not unexpected because with increasing (φ) fewer loads are applied along the length of helical ribs. This phenomenon decreases stiffness of lattice layer along (x) axis which is dependent on longitudinal stiffness of helical ribs.

But opposite to what was expected, E_y has a descending state as well. Because in fact, the amount of loads which are applied to horizontal ribs by helical ribs will increase when (φ) angle increases, and this finally causes additional strain of structure in (y) direction. This point causes decreases in structural stiffness in (y) direction while applying load.

G_{xy} has an increasing and descending state. This condition is similar to all other continuous plies. The maximum of this curve causes in $(\varphi = 45)$ which shows highest shear stiffness. At last, using the results obtained from elasticity curves, Poisson's ratio varieties versus (φ) can be determined easily.

Another result of this study is an isotropic condition in arrangement of ribs. For this state, according to mechanical properties of isotropic layers, modulus of elasticity in each direction should be the same. Therefore, according to the *Eqs*. (18 -19), if, $E_x = E_y$ (stiffness properties of isotropic layers) the following equation is derived:

$$\sin\varphi = 1/2\left(2 - \frac{E_c b_c / a_c}{E_h b_h / a_h}\right)^{1/2} \qquad (20)$$

and it shows relations between geometrical variables and mechanical properties of ribs in isotropic condition.

7. CONCLUSION

Superposition method is one of the simple methods used for analysis of lattice layers. In this study, using the method of deriving stiffness matrix of a lattice layer provides a simple approach for mechanical properties analysis of this kind of layers.

8. ACKNOWLEDGMENT

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